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About inclusion shape effect on the endurance limit of ductile matrix materials

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The theory describing fatigue mechanism in elastoplastic material containing pores or inclusions of different shape and size has been developed. The paper is a continuation of the analysis presented in *J. Mat. Sc.* **31** (1996) 2475 where the effect of only circular inclusion shape was investigated. An attempt at quantitative determination of the effect of endurance limit reduction by plastic zones size formed near the inclusions, and their cracking has been done. The geometrical configuration, consisting of round inclusion, horizontal, vertical and angular elliptical inclusions, from which a nucleating crack emerged, as well as sharp cracks was considered, and the stress intensity factors of such configurations were analysed. Based on threshold value of ΔK below which crack propagation ceases, the critical value of loading stress was determined for different shapes and sizes of pores using an equivalent ellipse concept. Theoretical results were compared with results from experiments, showing quite good agreement. For plastic zones size determination finite elements technique and photo-stress experimental method were applied. Surprisingly it was found that circular shape of the pores is the most dangerous which explains why so many earlier investigations using circular pore shape model are, so well supported by the experiment.

1. Introduction
Majority of models for the strength criteria of sintered materials assumes the circular shape of the pores. One such model developed earlier by the author of this paper was presented in previous publication [1, 2]. This model enables not only quantitative determination of the influence of matrix yield stress and size of inclusion but also, after modifications, the influence of the shape of this inclusion can be analysed.

The principal idea of the fracture process according to this model can be expressed as follows: Let us assume that an element of matrix containing separated pore of diameter D is loaded by the stress oscillations S acting far from the hole and yield stress of matrix is Y. This is shown in Fig. 1.

If $S * \alpha k > Y$, where αk-stress concentration factor, the plastic zones starts near the pore. By cyclic loading the plastic zone will very quickly form configuration presented in Fig. 2 containing hole with cracks. If the value of the stress intensity factor oscillations ΔK does not exceed the threshold value specific for any material and marked as $\Delta K\text{TH}$ the crack propagation will not occur. Thus the stability condition is

$$\Delta K\text{TH} > \Delta K \tag{1}$$

From this condition the stability stress oscillations S can be determined for a given value of $\Delta K\text{TH}$, yield stress Y of the matrix material and pore shape and diameter D. This stability stress oscillations S corresponds to the endurance limit of the material. Because fatigue fracture process usually starts from surface of elements where plastic deformation are facilitated the problem is restricted to the surface layer and consequently to plane stress condition.

The length of cracks in this problem, when compared to micro structural element size, are big enough to be treated according to linear elastic fracture mechanics rules according to the Kitagawa-Takahashi concept, see [1].

Only the biggest pores in the structure activate fracture process. The stress level in the considered problem is kept below the limit of 2/3 yield stress of the matrix material so the influence of the plastic zones in the crack tip is small.

2. Solution for circular inclusion
In assessing the fatigue integrity of a component, it is necessary to estimate the number of cycles required to produce a crack and subsequent number of cycles to propagate the crack to a limiting condition. It is clear

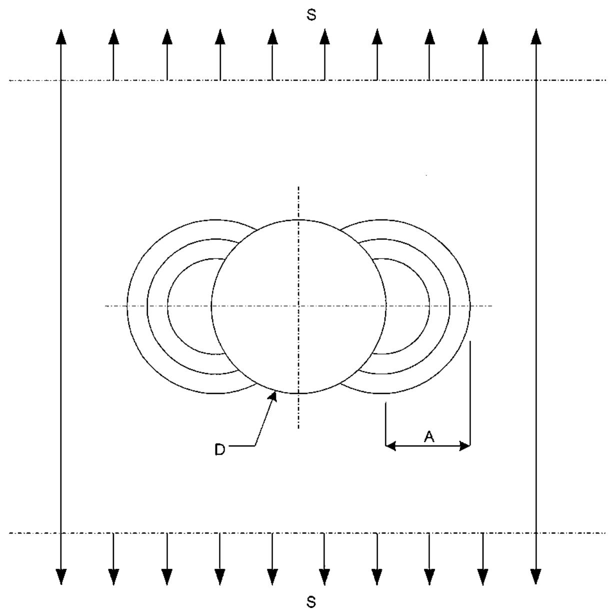

Figure 1 Plastic zone size.

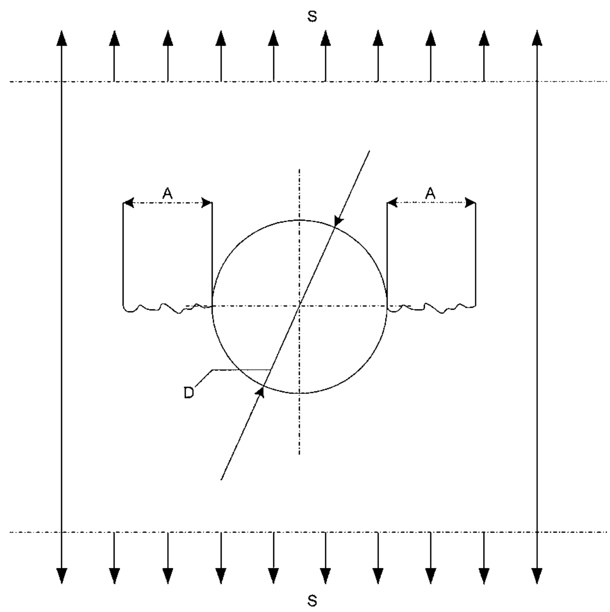

Figure 2 Fatigue crack growth.

that accumulation of plastic strain energy in a material will lead to crack formation. According to Coffin-Manson's law [3], the fatigue life N_f can be expressed by plastic strain component $\Delta\varepsilon_P$ as:

$$N_f = \left[\Delta\frac{\varepsilon_P}{\varepsilon_f}\right]^{\frac{1}{\alpha_1}} \tag{2}$$

where: ε_f and α_1, are coefficients.

From this formula it can be shown that the number of cycles for fracture is very small when plastic deformation occurs. For $\varepsilon_P = 10\%$, it is about 10 cycles only.

2.1. Plastic zone size

The quantitative analysis of plastic zone size around circular hole was performed numerically by Hinton [4], theoretically by Savin [5] and experimentally by Theocaris [6]. To continue analysis, a formula was proposed by author of this paper which approximate the theoretical results.

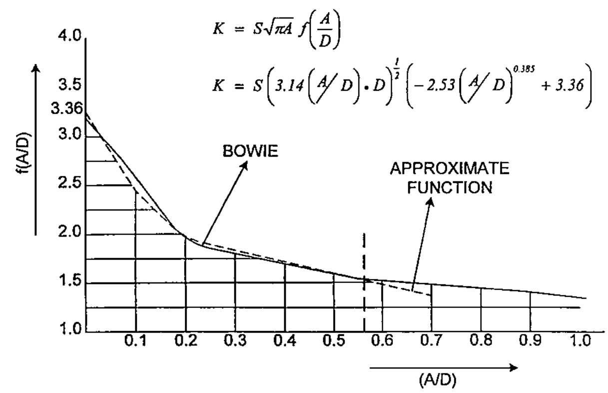

Figure 3 Stress intensity coefficient, approximate function.

$$A/D = \alpha[S/Y - \beta]^{\gamma} \tag{3}$$

where: A is the plastic zone range; D is the pore dimension; S is the applied stress oscillations value; Y is the yield stress of matrix material by cyclical loading and α, β, γ are coefficients ($\alpha = 2.58$; $\beta = 0.33$; $\gamma = 1.43$ for $0 < A/D < 0.55$).

Let us assume that an element of matrix containing separated circular pore of diameter D is loaded by stress oscillations S acting far from the hole and that yield stress of matrix is Y. This is shown in Fig. 1.

If $S * \alpha k > Y$, where αk-stress concentration factor the plastic zones starts near the pore.

By cyclic loading the plastic zone will crack quickly forming configuration presented in Fig. 2 containing circular hole with cracks.

2.2. *K*-factor determination

The stress intensity factor which describes the intensity of all stress components in the crack tip vicinity for the model under consideration was determined by Bowie [7]. The problem was solved using Mushelishvili's method [8] and stress function was assumed to have the form of polynomial. The results are presented in numerical and graphical form in Fig. 3. where

$$K = S^*\sqrt{\Pi^* A^*} f\left[\frac{A}{D}\right] \tag{4}$$

Bowie's results can be approximately expressed by the function

$$f\left[\frac{A}{D}\right] = \delta\left[\frac{A}{D}\right]^{\gamma} + \Theta \tag{5}$$

where: δ, γ and Θ are coefficients. $\delta = -2.53$; $\gamma = 0.385$; $\Theta = 3.36$; for $0 < A/D < 0.55$

$$K = S^*\sqrt{\Pi\left(\frac{A}{D}\right)D^*}\left[-2.53\left(\frac{A}{D}\right)^{(0.385)} + 3.36\right] \tag{6}$$

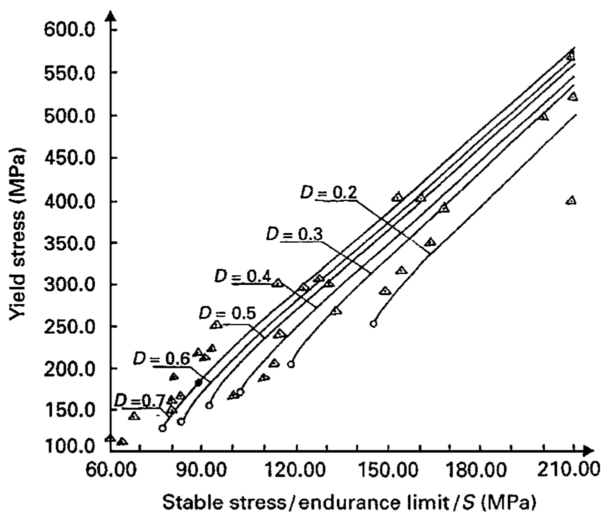

Figure 4 Stable stress (endurance limit) (MPa).

or

$$K = S^{*}\sqrt{2.58D\Pi\left(\frac{S}{Y}-0.333\right)^{1.43}} * \left\{-2.53\left[2.58\left(\frac{S}{Y}-0.333\right)^{1.43}\right]^{0.385}+0.336\right\} \quad (7)$$

If the value of the stress intensity factor K oscillation does not exceed the threshold value specified for any material and marked as ΔKTH the crack propagation will not occur. Thus the crack stability condition is $\Delta K\text{TH} > \Delta K$. From this condition the stability stress S can be determined for a given value of ΔKTH, yield stress Y, and pore diameter D. This stability stress oscilation S corresponds to the endurance limit of the material. The above developed theory was applied to porous powder metallurgy materials based on iron powder. The maximum observed pore size was 0.4 mm and pore sizes 0.2, 0.3, 0.4, 0.5, 0.6 and 0.7 mm were analysed. ΔKTH value determined in macro scale test was KTH, ΔKTH (MN* $m^{-3/2}$). This gives in micro scale, see [2] KTH, ΔKTH (MN* $m^{-3/2}$). The results of computer analysis are show in Fig. 4. Here the family of lines describing the relation between stability stress and yield stress of matrix for different inclusions D is presented at the background of experimental results of 30 different materials based on iron powder. Almost linear relation between stability stress S (endurance limit) and yield stress Y of matrix material was received from theoretical analysis of proposed model. Such linear relation is very well known for engineers working with powder metallurgy materials. Theoretical lines and experimental points are showing reflect able agreement. Relatively small influence of pore diameter D on stress S for big size of pores was revealed. The reduction of D from 0.7 mm to 0.3 mm results in the increase of S by 20 MPa only. The significant effect is noticed for small pores. This phenomena is also observed in practice of powder metallurgy. It can be mentioned in this place that the above presented concept of the fatigue process has found the support in experimental observations [9]. The formation of plastic zones emerging from graphite inclusions in spherical cast iron is clearly visible, also the cracks passing through this plastic zones can be noticed by microscopic observations.

3. Problem determination

The influence of non-circular shapes of pores existing in real material on the stability of fatigue cracks emerging from pores can also be investigated using the above described model. An alterative approaches to this problem can be found in [10–12].

The concept of an equivalent ellipse [13] can be used to analyze the influence of non-circular pore shape. According to this concept the stress distribution near the oval inclusion can be replaced by the stress distribution near a equivalent ellipse. The equivalent ellipse creates local stress field very similar to that produced by original inclusion. The principle is presented in Fig. 5. This ellipse is constructed so that its major axis has the same length as the respective dimension of the substituted shape and the radius of curvature, r, at the end of this axis is equal to the minimum radius of the curvature in the substituted hole. Application of this concept simplifies the problem and reduces the analysis to the elliptical shapes only. The method is useful for determination of stress distribution in the regions of maximum stress concentration were also formation of first plastic zones can be expected. The concept was successfully applied for stress concentration factors determination [14].

This paper is based on the assumption that equivalent ellipse idea can be extended to the early stages of plastic zones formation.

It is obvious that the results will be more accurate for more elliptical shapes of real pores. The shapes of the pores in real P.M. material (sintered Fe, NC 100-24 powder) were analysed [14] by the author of this paper. The results are presented in Fig. 6. From the diagram

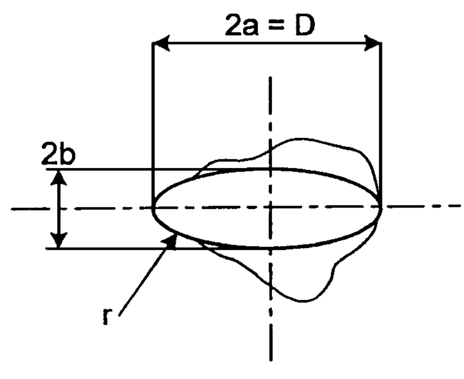

Figure 5 Equivalent ellipse application.

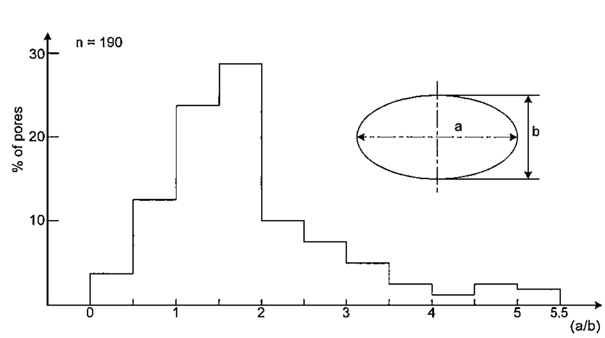

Figure 6 Pores shape statistical distribution.

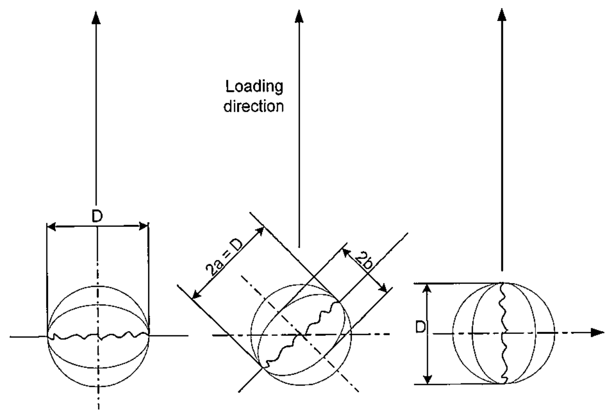

Figure 7 Equivalent ellipses variation.

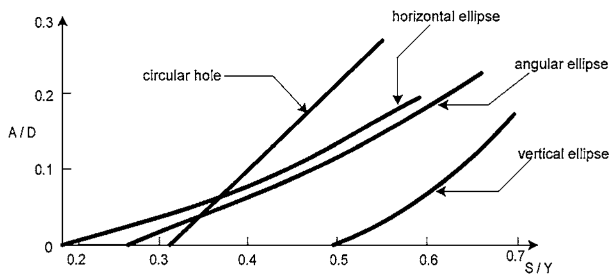

Figure 8 Range of plastic zones.

it can be observed that about 80% of the pore shapes can be replaced by ellipse with axis a/b ratio between 0.5 and 2, and that crack-like pores are not observed due to sintering action which rounds all sharp corners. Max noticeable value of a/b ratio was 5.5 only. In further analysis we assume that all pores were replaced by equivalent ellipses.

Such ellipses can vary in:

1. Max. diameter $D = 2a$;
2. The ratio a/b which locates all shapes between crack and circular hole, and
3. Inclination angle α in respect to loading direction Fig. 5.

According to Fig. 7 three elliptical shapes were investigated (horizontal, vertical and inclined at ($\alpha = 45°$) as well as sharp cracks corresponding to them. In addition the circular hole effect was analysed which is independent of inclination angle. The calculations are repeated for three diameters of pores i.e., $D = 0.4$, 0.6 and 0.8 mm indicating the same qualitative relations. The $D = 0.4$ mm corresponds with the maximum diameter of the pore observed in model material.

4. Calculation algorithm

The method of solution is the same as for circular shape. The first step of the analysis is the determination of the plastic zone size for each shape. Next the value of the stress intensity factor is determined based on the plastic zone range for certain loading stress S. Finally the value of the stress intensity factor is compared with threshold stress intensity factor KTH. From this the condition of stability is derived, and consequently the stable stress i.e., endurance limit is established.

The relation between plastic zone range A/D and applied stress S/Y have been obtained, using two methods independently, photo elasticity and finite elements both methods are in general agreement. It was also assumed that the plastic zone range for crack is negligibly small comparing with the crack length.

The relations of $A/D = f(S/Y)$ for different shapes of the pores are presented in Fig. 8. Continuing, we will analyze the value of stress intensity factors for: circle, differently oriented ellipses and differently oriented cracks. The calculation for the above mentioned configurations is done using stress intensity tables obtained from Murakami's works [5, 16]. The stress intensity factors are expressed in the form:

$$K_1 = F^* S^* \sqrt{\Pi\left(\frac{D}{2} + A\right)} \tag{8}$$

where: D is the maximum diameter of the pore; A is the length of the crack; F is the coefficient related to the geometry and, S is the applied stress.
The above formula can be transformed to:

$$K_1 = \left(\frac{Y}{100}\right)C \tag{9}$$

where:

$$C = 100\left(\frac{S}{Y}\right)F^* \sqrt{\Pi\left(\frac{D}{2} + A\right)} \tag{10}$$

To have the results independent of matrix yield stress, the C coefficient is introduced and analysed Fig. 9. The calculation is performed for three values of pore diameter ($D = 0.4$; $D = 0.6$ and $D = 0.8$ mm).

The method used for K calculations is as follows. For a chosen magnitude of S/Y, the value of A/D is found using graphical relation from Fig. 8. Next, the length of the crack A is determined for chosen value of pore diameter D. Knowing A and D the value of the coefficient F can be determined from tables placed in Stress Intensity Handbook [15].

Finally the C coefficient can be calculated as a function of (S/Y), for $D =$ const. (see Fig. 9).

For $K_1 = K\text{TH}$ the configuration is critical so the critical magnitude $C = C_{K\text{TH}}$ can be calculated from the formula

$$K\text{TH} = (Y/100)C_{K\text{TH}} \tag{11}$$

It is worth to notice that $C_{K\text{TH}}$ is a material dependent coefficient and like any C is expressed in ($\text{m}^{1/2}$) units. For model material the yield stress of matrix was $Y = 250$ MPa. For the $K\text{THM} \approx 3.65$ (MPa $^*\text{m}^{1/2}$) it gives $C_{K\text{TH}} \approx 1.46\ \text{m}^{1/2}$.

Now let's analyze the influence of crack-like pores. When the crack is directed perpendicularly to loading stress the calculation is elementary:

$$K_1 = \left(\frac{Y}{100}\right)C = S^* \sqrt{\Pi^* l} \tag{12}$$

where l is the crack length. So $C = f(S/Y)$ is a linear function and the lines representing this function are marked on the graphs Fig. 9 intersecting the $C_{K\text{TH}}$ level for S/Y magnitude corresponding with endurance

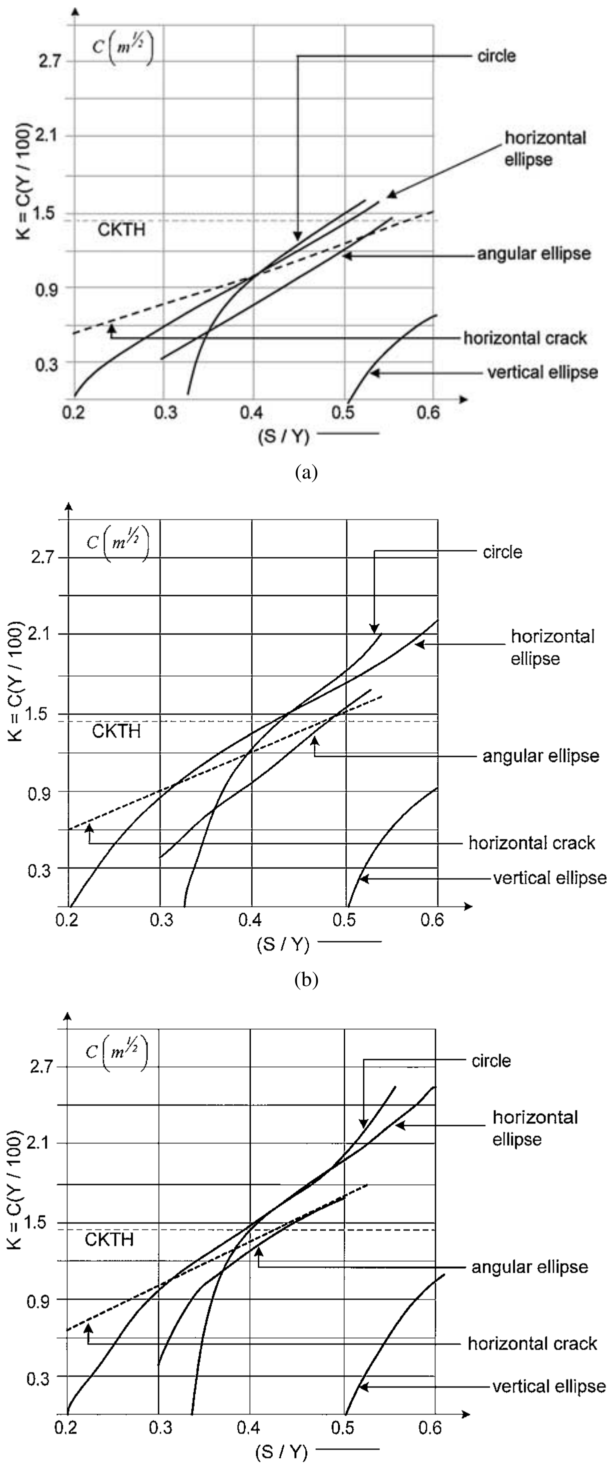

Figure 9 (a) Stress intensity factor for different pore shapes (dia = 0.4 mm); (b) stress intensity factor for different pore shapes (dia = 0.6 mm) and (c) stress intensity factor for different pore shapes (dia = 0.8 mm).

limit. However when the crack is inclined to the loading direction by angle α the mixed mode (I + II) criterion has to be applied [17, 18]. In this case the fracture activation stress S, for crack inclination angle ($\alpha \leq 45°$) is practically the same as for perpendicular to loading crack direction see Fig. 10.

$$S = G_\alpha * \frac{K_{\mathrm{IC}}}{\sqrt{\Pi l}}$$

where G_α is the inclination coefficient presented in Fig. 10a.

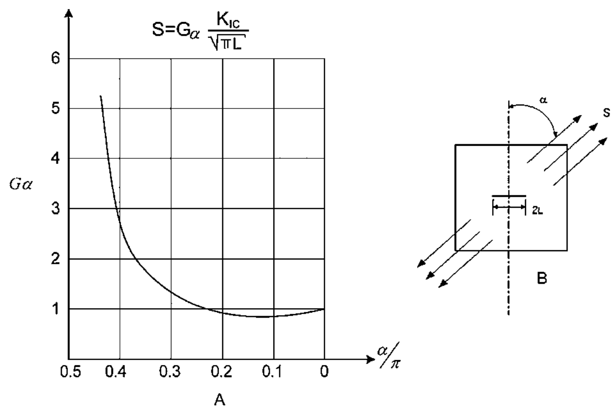

Figure 10 The influence of inclination angle α.

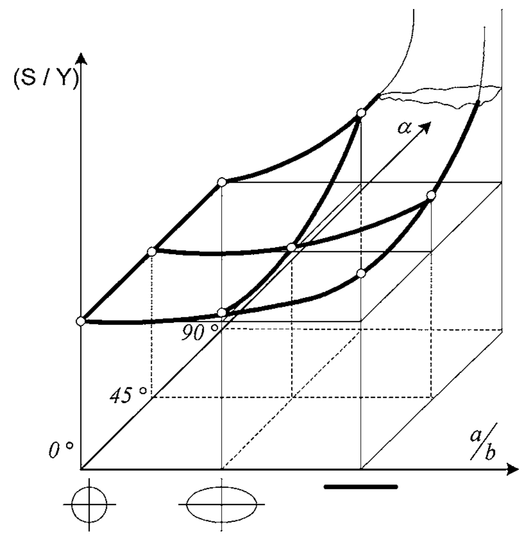

Figure 11 Response surface for stability stress.

For larger α angle this stress increases rapidly reaching infinity for vertical (parallel to loading) direction. So the crack-like pore will produce minimal critical stress when directed horizontally (perpendicularly to loading). Cracks of all other directions are less dangerous.

The effect of circular pores action is known from Section 2 of this work.

Thus presently we have nine points to build the response surface for the stability stress (S/Y) as function of the pore shape, they are: circle, ellipse and crack each of them for inclination angle a equal to 90°; 45° and 0°, see Fig. 11

5. Results

1. The analysis of the graphs in Fig. 9 shows directly the influence of pore size and shape on the stability stress (endurance limit).

2. It is visible that by increasing applied stress (S/Y) the critical value of $C_{K\mathrm{TH}}$ is reached first for circular shape and practically in the same point for horizontal ellipse. Horizontal crack, of identical length needs higher stress for instability. This surprising result is clear when we will remember that circular shape is forming big plastic zones which after cracking are enlarging K value.

3. The vertical (parallel to loading) ellipse are practically not active in fracture process and this can explain,

to certain extend, the effect of plastic deformation e.g., rolling and forging on the endurance limit when the pores can be converted into parallel to loading ellipses.

4. From the graphs in Fig. 9a–c the theoretical endurance limit for model material can be determined. Taking yield stress of the matrix material $Y = 250$ MPa, for pore diameter $D = 0.4$ mm and $D = 0.8$ mm the endurance limit will be located between 100 MPa and 120 MPa, (rounded pores of the size of 0.4 mm were directly observed at the surface of the sample). The endurance limit experimental values [1] (p. 136, Fig. 5.17) for model material are located between 70 MPa and 90 MPa for tension-compression test and for bending test this numbers are above 90 MPa and 120 MPa respectively showing a good agreement with the theory.

5. The above relation between circular, elliptical and sharp crack shape is preserved quantitatively for all three dimensions 0.4, 0.6 and 0.8 mm.

6. The moderate variation of the $C_{K\text{TH}}$ level (specially increase) will not introduce significant changes in above mentioned relations.

7. It is also interesting to notice that the proposed theory locates endurance limit in the region of 0.5–0.6 of yield stress [1, 19] which is known factor not only for powder metallurgy materials.

8. It is also very interesting to notice that the influence of the shape i.e., the difference in instability stress for circular shape and for crack of the some dimension, see Fig. 9, has the range of 20% for $D = 0.4$ mm and for bigger pores is even smaller (for 0.8 mm is in the range of 13%). When additionally we will remember that crack-like pores are not observed and the sharpest elliptical shape has $a/b \approx 5.5$, one can conclude that the influence of shape is very limited.

6. Conclusions

This work presents a model describing the behavior of stable fatigue cracks nucleated at pore-like defects in ductile matrix materials. The crack nucleus is assumed to have a length equivalent to the plastic zone size. The shape of real pores was replaced by equivalent ellipse and then plastic zone size was determined.

For circular pore the theoretical solutions for plastic zones size and for stress intensity factor were replaced by simplified functions which are easier to analyze.

From the diagram in Fig. 11 it is visible that the circular shape is most dangerous and such pores will cause fracture by minimal stress although one cannot exclude that in some materials the largest defects (high a/b ratio) may have a larger diameter-parameter D, than the largest round hole. The ellipse although activates plastic zones earlier will reach critical magnitude of C or K for higher stresses. Similarly sharp crack will reach the critical conditions for higher stress independently on angular orientation. The conclusion above justifies the series of already proposed models of fracture process where only circular inclusions were taken under consideration, despite different shapes of pores existing in real material. This also explains why so simple model used in the past had good correlation with experimental results.

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